Worcester County Mathematics League

Freshman Meet 1 – November 2, 2011

Round 1: Evaluation of Algebraic Expressions and Order of Operations

1

All answers must be in simplest exact form in the answer section

NO CALCULATOR ALLOWED

---

1. Let $a * b = b^a$. Find the value of the expression $(0 * 1) * (2 * 3)$.

2. Following the proper order of operations, evaluate:

$$7 \cdot \left[7 \cdot 7 - 7 \div 7 + (7 + 7) \div 7 \right]$$

3. Let $a \circ b$ represent the arithmetic mean of a and b. Evaluate the expression $\left(\left(\left(\frac{1}{2} \circ \frac{3}{4} \right) \circ \frac{5}{6} \right) \circ \frac{7}{8} \right) - \frac{3}{4}$ as a fraction reduced to lowest terms.

ANSWERS

(1 pt.) 1._____

(2 pts.) 2._____

(3 pts.) 3._____

Worcester County Mathematics League

Freshman Meet 1 – November 2, 2011

Round 2: Solving Linear Equations

2

All answers must be in simplest exact form in the answer section

NO CALCULATOR ALLOWED

---

1. If $\frac{1}{6}x = 15$, find the value of $\frac{4}{5}x$.

2. Solve for x: $0.27x - 4x + 8.925 = -0.4$

3. Solve for y: $\frac{2}{3}\left(3 - 5y\right) - \left[\frac{1}{5}\left(10 - 4y\right) - 5\right] = 12\frac{3}{5}$

ANSWERS

(1 pt.) 1._____

(2 pts.) 2._____

(3 pts.) 3._____

Worcester County Mathematics League

Freshman Meet 1 – November 2, 2011

Round 3: Logic Problems

3

All answers must be placed in the answer section at the bottom

NO CALCULATOR ALLOWED

1. It takes 5 seconds for a bell tower to strike 6 o'clock. If the bell strikes at exactly the beginning of the hour and the strikes are uniformly spaced, how many seconds does it take the tower to strike 12 o'clock?

3. On her birthday Athena was 14 years old and her mother was 41. Athena noticed that her age was the reverse of her mother's age. How old with Athena be the next time her age is the reverse of her mother's age?

4. Bart, Bert, Brit, and Burt play a game where the loser of a turn doubles the money of the other three players. After four turns they each have exactly $16. If each player lost exactly 1 turn each, how many dollars did the player with the most money start with?

ANSWERS

(1 pt.) 1. _____ seconds

(2 pts.) 2. _____ years old

(3 pts.) 3. _____ dollars

Worcester County Mathematics League

Freshman Meet 1 – November 2, 2011

Round 4: Ratio, Proportion and Variation

4

All answers must be in simplest exact form in the answer section

NO CALCULATOR ALLOWED

---

1. Every Thanksgiving Martha, who lives in Boston, visits her parents, who live in Youngstown, OH. Martha usually drives the 600 miles from Boston to Youngstown in 12 hours. If she were to increase her average speed by 10 miles per hour, how many hours would the trip take?

2. The exchange rate of the U.S. dollar (\$) to the Euro (€) is \$1 = 0.75€. If you go to Spain (which uses the Euro) with \$420 and you spend 255€, how much U.S. money would you return home with (in dollars)?

3. The volume V (in cubic meters, m^3) that a gas occupies varies directly as its temperature T (in degrees Kelvin, $°K$) and inversely as its pressure P (in kilograms per square meter, kg/m^2). Currently a certain gas has a volume of 20 m^3 at a pressure of 30 kg/m^2 and a temperature of $300°K$. If the temperature is increased by $20°K$ and the pressure is decreased by 10 kg/m^2, find the new volume of the gas (in cubic meters).

ANSWERS

(1 pt.) 1._____ hours

(2 pts.) 2. \$_____

(3 pts.) 3._____ m^3

Worcester County Mathematics League

Freshman Meet 1 – November 2, 2011

TEAM ROUND

All answers must *either* be in simplest exact form *or* as decimals rounded correctly to at least three decimal places **except for #6** (3 pts. each)

APPROVED CALCULATORS ALLOWED

1. Find the smallest two-digit number that satisfies the following five conditions:
   - The number is not an odd number
   - The number has exactly four factors (including 1 and itself)
   - If you reverse the digits of the number, a two-digit prime number is formed
   - The sum of the digits is a two-digit prime number
   - One of the digits of the number is a perfect square

2. In baseball, a ***batting average*** is the number of hits divided by the number of at-bats. While checking the batting average of my favorite player I noticed that if he had twice the number of hits, his average would be 1.4 times as great as if his at-bats were reduced by the number of his hits. Assuming that it is not zero, what is his batting average?

3. Compute the result of the following calculation: twelve more than the positive square root of the sum of one and the square root of 64 all divided by five.

4. Dana can run around an oval track once in 40 seconds. Emily, running in the opposite direction, meets Dana every 15 seconds. How many seconds does it take for Emily to run around the track once?

5. When written as a fraction reduced to lowest terms, the solution of the equation $\frac{1}{2}x-\frac{2}{3}x+\frac{3}{4}x-\frac{4}{5}x=x-\frac{5}{6}$ is $x=\frac{a}{b}$. Find the sum $a+b$.

6. The grade you earn in math is directly proportional to the amount of time that you spend studying. The amount of time that you spend studying is inversely proportional to the amount of time that you spend playing video games. You know that if you spend 1.5 hours studying, you'll earn a 75% and if you spend 2 hours studying, you can play video games for 3.5 hours. How much time can you play video games if you want to earn a 90% in math? On the answer sheet, express your answer as *hours* : *minutes*, to the nearest minute.

7. Elliot the bricklayer would take 9 hours to build a certain wall on his own, while his assistant Manny would take 10 hours to build the same wall. If the two work together, they stop for a chat so that they lay a total of 10 fewer bricks per hour. If it takes them 5 hours to build the wall together, how many bricks are in the wall?

8. In how many different ways can 35 cents be made using any combination of quarters, dimes, nickels and pennies?

Worcester County Mathematics League

Freshman Meet 1 – November 2, 2011

ANSWERS

Round 1

1. 9

2. 350

3. $\frac{5}{96}$ (only)

Round 2

1. 72

2. $2.5 = 2\frac{1}{2} = \frac{5}{2}$

3. –3

Round 3

1. 11

2. 25

3. 33

Round 4

1. 10

2. 80

3. 32

Team Round

1. 74

2. $0.3 = \frac{3}{10}$

Note: OK to accept 0.300 since that is conventional for batting averages

3. 3

4. 24

5. 123

6. 3:53

7. 900

8. 24

Worcester County Mathematics League

Freshman Meet 1 – November 2, 2011

BRIEF SOLUTIONS

Round 1

1. $(0*1)*(2*3) = 1^0 * 3^2 = 1*9 = 9^1 = 9$

2. $7\cdot\left[7\cdot 7 - 7 \div 7 + (7+7) \div 7\right] = 7(49 - 1 + 14 \div 7) = 7(48+2) = 7\cdot 50 = 350$

3. $\left(\left(\left(\frac{1}{2}\circ\frac{3}{4}\right)\circ\frac{5}{6}\right)\circ\frac{7}{8}\right) - \frac{3}{4} = \left(\left(\left(\frac{\frac{1}{2}+\frac{3}{4}}{2}\right)\circ\frac{5}{6}\right)\circ\frac{7}{8}\right) - \frac{3}{4} = \left(\left(\frac{5}{8}\circ\frac{5}{6}\right)\circ\frac{7}{8}\right) - \frac{3}{4} = \left(\left(\frac{\frac{5}{8}+\frac{5}{6}}{2}\right)\circ\frac{7}{8}\right) - \frac{3}{4} = \left(\frac{35}{48}\circ\frac{7}{8}\right) - \frac{3}{4}$

$= \left(\frac{35}{48}\circ\frac{7}{8}\right) - \frac{3}{4} = \frac{\frac{35}{48}+\frac{7}{8}}{2} - \frac{3}{4} = \frac{77}{96} - \frac{3}{4} = \frac{77}{96} - \frac{72}{96} = \frac{5}{96}$.

Round 2

1. $\frac{1}{6}x = 15 \Rightarrow x = 6\cdot 15 = 90$. Next, $\frac{4}{5}\cdot 90 = 4\cdot 18 = 72$.

2. One way involves multiplying both sides by 100: $0.27x - 4x + 8.925 = -0.4 \Rightarrow 27x - 400x + 892.5 = -40$. Solving for x yields: $27x - 400x + 892.5 = -40 \Rightarrow 373x = 932.5 \Rightarrow x = 2.5$.

3. Multiply both sides by 15, distribute and solve for y: $\frac{2}{3}\left(3-5y\right) - \left[\frac{1}{5}\left(10-4y\right) - 5\right] = 12\frac{3}{5}$

$\Rightarrow 10(3-5y) - 3(10-4y) + 75 = \frac{63}{5}\cdot 15 \Rightarrow 30 - 50y - 30 + 12y = 189 - 75 \Rightarrow -38y = 114 \Rightarrow y = -3$.

Round 3

1. When a clock strikes 6 o'clock, there are 5 time intervals between the first strike and the last strike. Each of these strikes takes 1 second. When the clock strikes 12 o'clock, there are 11 time intervals between the first strike and the last strike. Since each of these take 1 second, it take the clock 11 seconds to strike 12 o'clock.

2. Let the mother's age be $10T + U$ and let Athena's age be $10U + T$. Athena is currently $41 - 14 = 27$ years younger than her mom, so that $10T + U = 10U + T + 27 \Rightarrow T - U = 3$. Currently $T = 4$ and $U = 1$, which means next time $T = 5$ and $U = 2 \Rightarrow$ Athena will be 25.

3. Work backwards from the fact that all four end up with \$16. Here is a table summarizing that method, ignoring the order of the losses:

After the 4th loss	16 (loser)	16	16	16
After the 3rd loss	40	8 (loser)	8	8
After the 2nd loss	20	36	4 (loser)	4
After the 1st loss	10	18	34	2 (loser)
Beginning	5	9	17	**33**

Round 4

1. Let $x =$ the time it will take at the new speed. First, $\frac{600 \text{ miles}}{12 \text{ hours}} = 50$ mph so that her new speed is 60 mph. We have

$60 \text{ mph}\cdot x \text{ hours} = 600 \text{ miles} \Rightarrow x = 10 \text{ hours}$.

2. You started with $\$420 = 420 \cdot (0.75) = 315€$. You spent 255€, leaving you with 60€. Converting back to dollars gives $60 \cdot \frac{4}{3} = \$80$.

3. Let V, T, and P = the volume, temperature, and pressure of the gas, respectively. We have $V = k\frac{T}{P}$, where k = the constant of proportionality. The given information yields: $20 = k \cdot \frac{300}{30} \Rightarrow k = 2$ so that $V = \frac{2T}{P}$. Now, the new volume is $V = \frac{2(320)}{20} = 32$.

Team Round

1. At least one of the digits will be a 1, 4 or 9. An even number with 4 factors will have the form $2p$, where p is a prime number greater than 2. There are 4 numbers satisfying the previous two properties: 14, 34, 74, and 94. These are all reversible into a prime except for 94, but the only one whose digital sum is a prime is 74.

2. Let x = the number of hits achieved and y = the number of at-bats so that his batting average is $\frac{x}{y}$. If he had 2 times the hits, his batting average would be $\frac{2x}{y}$, which is $\frac{7}{5}$ times his batting average if his at-bats were only $y - x$. This gives: $\frac{2x}{y} = \frac{7}{5} \cdot \frac{x}{y-x} \Rightarrow 2\left(\frac{x}{y}\right) = \frac{\frac{7}{5}\left(\frac{x}{y}\right)}{1-\left(\frac{x}{y}\right)}$. Now let $A = \frac{x}{y}$, the batting average. We have $2A = \frac{1.4A}{1-A} \Rightarrow 2A - 2A^2 = 1.4A$ $\Rightarrow 0.6A - 2A^2 = 0 \Rightarrow A(0.6 - 2A) = 0 \Rightarrow A = 0$ or $A = 0.3$. Taking the non-zero result, the answer is 0.3.

3. Twelve more than the positive square root of the sum of one and the square root of 64 all divided by five = $\frac{12 + \sqrt{1 + \sqrt{64}}}{5} = \frac{12 + \sqrt{9}}{5} = \frac{15}{5} = 3$.

4. Let t = Emily's time. In 15 seconds Dana completes $\frac{15}{40}$ of a lap, while Emily completes $\frac{15}{t}$ of a lap. Since they complete a one lap combined in 15 seconds we have $\frac{15}{40} + \frac{15}{t} = 1 \Rightarrow \frac{15}{t} = \frac{25}{40} = \frac{5}{8} \Rightarrow t = 24$ seconds.

5. Multiply both side by 60, combine like terms and solve for x: $\frac{1}{2}x - \frac{2}{3}x + \frac{3}{4}x - \frac{4}{5}x = x - \frac{5}{6} \Rightarrow$ $30x - 40x + 45x - 48x = 60x - 50 \Rightarrow 73x = 50 \Rightarrow x = \frac{50}{73} \Rightarrow a + b = 50 + 73 = 123$.

6. Let M, S, and V = your math score, time studying, and time spent playing video games respectively, so that $M = pS$ and $S = \frac{q}{V}$, where p and q are constants. From the first piece of given info, $75 = p \cdot 1.5 \Rightarrow p = 50$ so that $M = 50S$. Next, $2 = \frac{q}{3.5} \Rightarrow q = 7$ so that $S = \frac{7}{V}$. Substituting $S = \frac{7}{V}$ into $M = 50S$ gives $M = \frac{350}{V} \Rightarrow V = \frac{350}{M}$. If you want a score of 90, you can play video games for $\frac{350}{90} = 3.889$ hours or 3 hours and $0.889 \cdot 60 = 53$ minutes.

7. Suppose Elliot lays x bricks per hour on his own, and Manny lays y bricks per hour. Then the number of bricks in the wall is $9x$ which is equal to $10y$. Working together they lay $x + y - 10$ bricks per hour, so the number of bricks in the wall is also $5(x + y - 10)$. Solving the equations $9x = 5(x + y - 10)$ and $10y = 5(x + y - 10)$ simultaneously yields $x = 100$ and $y = 90 \Rightarrow$ the number of bricks in the wall is 900.

8. Make a systematic list:

Quarters (25)	Dimes (10)	Nickels (5)	Pennies (1)
1	1	0	0
1	0	2	0
1	0	1	5
1	0	0	10
0	3	1	0
0	3	0	5
0	2	3	0
0	2	2	5
0	2	1	10
0	2	0	15
0	1	5	0
0	1	4	5
0	1	3	10
0	1	2	15
0	1	1	20
0	1	0	25
0	0	7	0
0	0	6	5
0	0	5	10
0	0	4	15
0	0	3	20
0	0	2	25
0	0	1	30
0	0	0	35

There are 24 possible combinations.